A Derivation of Demand Equations and Price Indices

The home-country consumer's problem is to maximize expected utility,

$$E[U(C,l)],$$

subject to

$$\int_0^n p_H(i)c_H(i)di + \int_n^1 p_F(i)c_F(i)di = w(1-l) + \pi$$

where

$$U(C,l) = \frac{1}{1-\rho}C^{1-\rho} + \eta l.$$

C is a general consumption index over both goods of Cobb-Douglas form,

$$C = \frac{c_H^n c_F^{1-n}}{n^n(1-n)^{1-n}}.$$

There are five tasks in solving the consumers problem, which are separated and approached in the following order:

(1) finding the overall demand for domestic and foreign goods as a group,

(2) deriving the general price index,

(3) deriving the separate price indices for each country's basket of goods produced in the home country,

(4) finding the demand for each specific good, and

(5) deriving the consumer's wage relation.

A.1 Deriving the overall demand for home and foreign goods

First, to find the home consumer's demand for each of the two country's goods as a fraction of home-country expenditure, the Langrangian equation is specified as

$$L = \max E[\frac{1}{1-\rho}\left(\frac{c_H^n c_F^{1-n}}{n^n(1-n)^{1-n}}\right)^{1-\rho} + \eta l] + \lambda E\left[w(1-l) + \pi - p_H c_H - p_F c_F\right].$$

Defining $\gamma = \frac{1}{n^n(1-n)^{1-n}}$, first order conditions are

$$\frac{\partial L}{\partial c_H} : \gamma n C^{-\rho} c_H^{n-1} c_F^{1-n} - \lambda p_H = 0 \quad (1)$$
$$\frac{\partial L}{\partial c_F} : \gamma(1-n) C^{-\rho} c_H^{n} c_F^{-n} - \lambda p_F = 0 \quad (2)$$
$$\frac{\partial L}{\partial \lambda} : w(1-l) + \pi - p_H c_H - p_F c_F = 0 \quad (3)$$

Dividing (1) by (2) yields the relation

$$c_H = \left(\frac{n}{1-n}\right)\left(\frac{p_F}{p_H}\right) c_F. \tag{4}$$

Calling nominal income, $w(1-l)+\pi$, Y and substituting (4) into (3), demand for each country's goods emerge as a fraction of the home consumer's expenditure:

$$\begin{aligned} p_H \left[\left(\frac{n}{1-n}\right)\left(\frac{p_F}{p_H} c_F\right)\right] + p_F c_F &= Y \\ p_F c_F \left[1 + \left(\frac{n}{1-n}\right)\right] &= Y \\ p_F c_F &= (1-n)Y \qquad (5) \\ \text{(from (4) and (5))} \quad p_H c_H &= nY. \qquad (6) \end{aligned}$$

A.2 Deriving the general price index

Define P as the amount of home country currency necessary to purchase one unit of the consumption basket C. Then to derive P, the general price index, we set

$$\gamma c_H^n c_F^{1-n} \equiv 1$$

and substitute demand equations (5) and (6) (slightly rearranged to isolate c_H and c_F) for c_H and c_F:

$$\gamma \left(\frac{nY}{p_H}\right)^n \left(\frac{(1-n)Y}{p_F}\right)^{1-n} = 1. \tag{7}$$

Since the general price index is the amount of a consumer's nominal income required to purchase one unit of C, Y is replaced by P in (7) and emerges as a function of the price indexes of each country's individual basket of goods produced in the home country.

$$\begin{aligned} \gamma \left(\frac{nP}{p_H}\right)^n \left(\frac{(1-n)P}{p_F}\right)^{1-n} &= 1 \\ P\left(\frac{1}{p_H^n p_F^{1-n}}\right) &= 1 \\ P &= p_H^n p_F^{1-n} \end{aligned} \tag{8}$$

A.3 Deriving separate price indexes for the home and foreign basket of goods

To derive the price index for the home-good and the foreign-good basket of products offered in the home country, it is necessary to solve a second maximization problem. Now, expenditure on each basket is minimized to find the amount necessary to purchase one unit of the consumption index over each country's goods. To illustrate, the price index, p_H is derived here.

As described in Section 2.1, the separate consumption index over home goods is defined by

$$c_H = \left[n^{-\frac{1}{\mu}} \int_0^n c_H(i)^{\frac{\mu-1}{\mu}} di \right]^{\frac{\mu}{\mu-1}}.$$

To find the minimum expenditure required to purchase one unit of c_H, the consumer solves the problem

$$\min \int_0^n p_H(i) c_H(i) di$$

subject to the constraint that only one unit of c_H is purchased. The Langrange now takes the form

$$L = \min \int_0^n p_H(i) c_H(i) di + \lambda \left[1 - \left[n^{-\frac{1}{\mu}} \int_0^n c_H(i)^{\frac{\mu-1}{\mu}} di \right]^{\frac{\mu}{\mu-1}} \right].$$

It is useful to note here that the Lagrange muntiplier, λ, is a proxy for the amount required to buy one (extra) unit of c_H, the basket of home goods. Therefore, by solving for λ, one is solving for the home-goods price index, p_H.

First-order conditions are

$$\frac{\partial L}{\partial c_H(i)} \quad : \quad p_H(i) - \lambda \left(\frac{\mu}{\mu-1}\right) \left(n^{-\frac{1}{\mu}} \int_0^n c_H(i)^{\frac{\mu-1}{\mu}} di\right)^{\frac{1}{\mu-1}} \left(\frac{\mu-1}{\mu}\right) n^{-\frac{1}{\mu}} c_H(i)^{-\frac{1}{\mu}} = 0 \tag{9}$$

$$\frac{\partial L}{\partial \lambda} \quad : \quad 1 - \left[n^{-\frac{1}{\mu}} \int_0^n c_H(i)^{\frac{\mu-1}{\mu}} di\right]^{\frac{\mu}{\mu-1}} = 0 \tag{10}$$

Recognizing that $\left(n^{-\frac{1}{\mu}} \int_0^n c_H(i)^{\frac{\mu-1}{\mu}} di\right)^{\frac{1}{\mu-1}} = c_H^{\frac{1}{\mu}}$ and rearranging, one can isolate $c_H(i)$ in equation (9).

$$\begin{aligned} p_H(i) &= n^{-\frac{1}{\mu}} \lambda c_H^{\frac{1}{\mu}} c_H(i)^{-\frac{1}{\mu}} \\ n^{\frac{1}{\mu}} \left(\frac{1}{\lambda}\right) c_H^{-\frac{1}{\mu}} p_H(i) &= c_H(i)^{-\frac{1}{\mu}} \\ c_H(i) &= \left(\frac{1}{n}\right) \lambda^{\mu} c_H p_H(i)^{-\mu} \end{aligned}$$

Noting again that $c_H \equiv 1$,

$$c_H(i) = \left(\frac{1}{n}\right) \lambda^{\mu} p_H(i)^{-\mu}. \tag{11}$$

Substituting (11) into (10), one can solve for λ:

$$
\begin{aligned}
\left\{ n^{-\frac{1}{\mu}} \int_0^n \left[\left(\frac{1}{n}\right) \lambda^{\mu} p_H(i)^{-\mu} \right]^{\frac{\mu-1}{\mu}} di \right\}^{\frac{\mu}{\mu-1}} &= 1 \\
\left[n^{-\frac{1}{\mu} - \left(\frac{\mu-1}{\mu}\right)} \lambda^{\mu\left(\frac{\mu-1}{\mu}\right)} \int_0^n p_H(i)^{-\mu\left(\frac{\mu-1}{\mu}\right)} di \right]^{\frac{\mu}{\mu-1}} &= 1 \\
\left[\left(\frac{1}{n}\right) \lambda^{\mu-1} \int_0^n p_H(i)^{1-\mu} di \right]^{\frac{\mu}{\mu-1}} &= 1 \\
\left[\left(\frac{1}{n}\right) \int_0^n p_H(i)^{1-\mu} di \right]^{\frac{\mu}{\mu-1}} &= \lambda^{-\mu} \\
\lambda &= \left[\left(\frac{1}{n}\right) \int_0^n p_H(i)^{1-\mu} di \right]^{\frac{1}{1-\mu}} = p_H.
\end{aligned}
$$

In the same way, p_F is found to be

$$
p_F = \left[\left(\frac{1}{1-n}\right) \int_n^1 p_F(i)^{1-\mu} di \right]^{\frac{1}{1-\mu}}.
$$

Analogous price indexes result for the market in the foreign country, since preferences are identical.

A.4 Deriving the demand for individual home-country goods

From (11), it is evident that when $c_H = 1$,

$$\lambda = n^{\frac{1}{\mu}} c_H(i)^{\frac{1}{\mu}} p_H(i) = p_H$$

and

$$c_H(i) = \frac{1}{n}\left(\frac{p_H(i)}{p_H}\right)^{-\mu}. \tag{12}$$

Becuase preferences over the continuum of home goods are homothetic, the amount of each home good ($c_H(i)$) consumed as a fraction of all home goods (c_H) consumed by home-country residents is constant. Thus, equation (12) implies that the demand for each unique home good can be generalized to

$$c_H(i) = \frac{1}{n}\left(\frac{p_H(i)}{p_H}\right)^{-\mu} c_H$$

when $c_H \neq 1$.[1]

[1]The solution methods used in this appendix for the consumer's problem are outlined in Obstfeld and Rogoff (1998, Chapters 4.1 and 4.5).

A.5 Deriving the consumer's wage relation

Using the price indexes and demand equations derived above, $p_H c_H + p_F c_F = PC$. The consumer's Lagrangian equation can be rewritten in the form

$$L = \max E[\frac{1}{1-\rho}\left(\frac{c_H^n c_F^{1-n}}{n^n(1-n)^{1-n}}\right)^{1-\rho} + \eta l] + \lambda E\left[w(1-l) + \pi - PC\right].$$

First order conditions are

$$\begin{aligned} \frac{\partial L}{\partial C} &: \quad C^{-\rho} - \lambda P = 0 \Rightarrow \lambda = \frac{1}{PC^{\rho}} \\ \frac{\partial L}{\partial l} &: \quad \eta - \lambda w = 0 \end{aligned}$$

Combining the two first order conditions yields the consumer's wage relation,

$$w = \frac{\eta}{\lambda} = \frac{\eta P}{C^{-\rho}}$$

B Derivation of the Firm's Pricing Rules

In the absence of uncertainty, the monopolistically competitive firm would maximize nominal profits, without regard to the covariance of profits and marginal utility. Since there would be only one state, which presumably both the firm and its owners could foresee, there is no need to introduce a stochastic discount factor into the problem to weight expected profits by the marginal utility (well-being) of the consumer-owners in each possible state.

Therefore, the firm would maximize nominal profits[2],

$$\pi = (p_H(i) - w)\, c_H(i) + S\, (p_H^*(i) - w^*)\, c_H^*(i) - F$$

First-order conditions for a price-setting firm are

$$\frac{\partial \pi}{\partial c_H(i)} \quad : \quad p_H(i) + \frac{\partial p_H(i)}{\partial c_H(i)} c_H(i) - w = 0 \tag{13}$$

$$\frac{\partial \pi}{\partial c_H^*(i)} \quad : \quad S\left(p_H^*(i) + \frac{\partial p_H^*(i)}{\partial c_H^*(i)} c_H^*(i)\right) - S w^* = 0 \tag{14}$$

Extracting $\frac{1}{p_H(i)}$ from the first two terms of the left-hand side of (13), the expression can be simplified

$$p_H(i)\left[1 + \left(\frac{\partial p_H(i)}{\partial c_H(i)}\right)\left(\frac{c_H(i)}{p_H(i)}\right)\right] \quad = \quad w$$

[2]Here, the linear technology,

$$\begin{aligned} L_H(i) &= c_H(i) \\ L_H^*(i) &= c_H^*(i), \end{aligned}$$

is used to substitute the quantity produced (which in the segmented markets equals the quantity consumed) for the labor input. For the case of perfect foresight, the productivity shock is omitted.

$$p_H(i)\left[1+\frac{1}{e}\right] = w,$$

where e is the price-elasticity of demand for $c_H(i)$. Due to the CES preferences specified for the home-goods basket above, $e = -\mu$, so that

$$\begin{aligned} p_H(i)\left[1-\frac{1}{\mu}\right] &= w \\ p_H(i) &= \left(\frac{\mu}{\mu-1}\right) w. \end{aligned}$$

Similarly, the foreign price is set such that

$$p_H^*(i) = \left(\frac{\mu}{\mu-1}\right) w^*. \tag{15}$$

The firm in this way sets prices as a markup over the wage in each country, where the markup is a function of the price-elasticity of demand for its product. Interestingly, in the unrestricted case, the exchange rate cancels out and has no effect on the home firm's pricing in the foreign market. However, the firm must also operate in the overseas market subject to the combined entry/zero-profit (market equilibrium) conditions,[3]

[3]This process of constrained optimization under potential entry follows the methodology of Dixit and Stiglitz (1977, pp. 299-300).

$$S\left(p_H^*(i) - w^*\right) c_H^*(i) - F = 0. \tag{16}$$

Rearranging this constraint, one can see that price must also be governed by the equation

$$p_H^*(i) = \frac{F + Sw^* c_H^*(i)}{Sc_H^*(i)} = \left(\frac{1}{S}\right)\left(\frac{F}{c_H^*(i)}\right) + w^*. \tag{17}$$

That is to say, the price must be set just high enough to cover the average fixed cost of operating in the overseas market, as well as the marginal cost.[4] Otherwise, the home firm will choose not to produce in the foreign country. If the price is any higher than in (17), positive profits will result. In this case, rival home firms will be enticed to set up plants overseas, until all firms must lower their prices enough to push profits back to zero. It is important to note that the exchange rate does not cancel out, opening the door for exchange rate volatility to impact pricing and the scale of production when uncertainty is introduced. In section B.1, uncertainty is introduced in the form of shocks to productivity and the money supply, as in Section 2.2 of the main text. Equations analogous to (15) and (17) are combined and

[4]In a partial equilibrium setting, the home firm's problem ends here, since the wage is exogenous. The firm will set a price corresponding to equation (15) as long as the markup, $\frac{\mu}{\mu-1}$, is sufficiently high to satisfy the condition in (17). Otherwise, the home firm will not produce at all in the foreign country.

solved to find a reduce form for the home firm's pricing rule in the foreign market.

B.1 Introducing uncertainty into the firm's problem

As explained in Section 2.2, when uncertainty is introduced, the firm will maximize the *market value* of nominal profits. The market value of nominal profits accounts for the covariance of revenues with the welfare of the firm's owners. Because consumers are the firm's owners, marginal utility is used to express their well-being under any given realization of the monetary and productivity shocks. The firm's problem under uncertainty becomes

$$\max_{c_H(i), c_H^*(i)} E[U_c \pi]$$

where[5]

$$\pi = \left(p_H(i) - \frac{1}{\theta}w\right) c_H(i) + S\left(p_H^*(i) - \frac{1}{\theta^*}w^*\right) c_H^*(i) - f.$$

[5]Here, the productivity shocks are included in the equations defining the firm's technology,

$$\begin{aligned} \theta L_H(i) &= c_H(i) \\ \theta^* L_H^*(i) &= c_H^*(i). \end{aligned}$$

The term f is a fixed cost the home firm must pay to produce in the foreign market, denominated in the home currency. When paid, the fixed cost can be denominated in the foreign currency, but it enters the home firm's decision-making process converted at some initial exchange rate, which is embedded (as a constant) in the value f.

First order conditions are

$$\frac{\partial \pi}{\partial c_H(i)} : E\left[U_c\left(p_H(i) + \frac{\partial p_H(i)}{\partial c_H(i)} c_H(i) - \frac{1}{\theta} w\right)\right] = 0 \tag{18}$$

$$\frac{\partial \pi}{\partial c_H(i)} : E\left[U_c\left(p_H^*(i) + \frac{\partial p_H^*(i)}{\partial c_H^*(i)} c_H^*(i) - \frac{1}{\theta^*} w^*\right)\right] = 0 \tag{19}$$

Parallel to the case of perfect foresight above, equations (18) and (19) simplify to unrestricted pricing rules stating that price will be set as a markup over the risk-weighted wage (marginal cost) in each country:

$$p_H^*(i) = \left(\frac{\mu}{\mu - 1}\right) \frac{E[U_c S \frac{w^*}{\theta^*}]}{E[U_c S]} \tag{20}$$

$$p_H(i) = \left(\frac{\mu}{\mu - 1}\right) \frac{E[U_c \frac{w}{\theta}]}{E[U_c]}. \tag{21}$$

Analogous unrestricted pricing rules emerge for the foreign firm:

$$p_F(i) = \left(\frac{\mu}{\mu - 1}\right) \frac{E[U_{c^*} \frac{w}{S\theta}]}{E[U_{c^*}(\frac{1}{S})]} \tag{22}$$

$$p_F^*(i) \quad = \quad \left(\frac{\mu}{\mu-1}\right)\frac{E[U_{c^*}\frac{w^*}{\theta^*}]}{E[U_{c^*}]} \tag{23}$$

However, the home firm must still make sure that the price it sets in the foreign market will be high enough to both cover its fixed costs. Thus, the price must satisfy the entry condition,

$$E\left[U_c\left(S\left(p_H^*(i)-\frac{1}{\theta^*}w^*\right)c_H^*(i)\right)\right]-f\geq 0.$$

The foreign firm has a similar entry condition,

$$E\left[U_{c^*}\left(\frac{1}{S}\left(p_F(i)-\frac{1}{\theta}w\right)c_F(i)\right)\right]-f^*\geq 0.$$

In the interests of tractability in the following analysis, it is assumed that the fixed cost incurred by firms investing overseas is just large enough that these conditions are binding– that is, that expected variable profits are exactly equal to the fixed costs, as in Christiano, Eichenbaum, and Evans (2001). Rearranging, it is evident that the home firm must set its price in the foreign market according to (21) such that it satisfies the relation

$$p_H^*(i)=\frac{E[U_cS\frac{w^*}{\theta^*}c_H^*(i)]+f}{E[U_cSc_H^*(i)]} \tag{24}$$

and the foreign firm must set its price such that

$$p_F(i) = \frac{E[U_{c^*}\frac{w}{S\theta}c_F(i)] + f^*}{E[U_{c^*}(\frac{1}{S})c_F(i)]} \tag{25}$$

To examine the effect of exchange-rate volatility on the foreign firm's pricing behavior in the home market, it is necessary to combine (22) and (25), make appropriate substitutions for U_c, w^*, and $c_H^*(i)$ and solve for a reduced form. Setting (22) equal to (25) implies that the firm will set prices to cover its total costs subject to the behavior of the demand curve embedded in the unrestricted pricing rule. Thus, we have

$$\left(\frac{\mu}{\mu-1}\right)\frac{E[U_{c^*}\frac{w}{S\theta}]}{E[U_{c^*}(\frac{1}{S})]} = \frac{E[U_{c^*}\frac{w}{S\theta}c_F(i)] + f^*}{E[U_{c^*}(\frac{1}{S})c_F(i)]}. \tag{26}$$

A key feature of the model Devereux and Engel 2000 that prevents changes in the foreign money supply from impacting home consumption is factor-price equalization ($w = Sw^*$). Factor-price equalization emerges in their model as a result of a complete set of state-contingent bonds. Since this is a one-period model, intertemporal borrowing and lending is not possible. To show that it is the fixed cost driving the new result in this simple model, it is assumed that labor is perfectly mobile, also producing factor-price equalization.

Making the appropriate substitutions using the demand and wage equations, (26) becomes

$$\frac{\alpha E[\left(\frac{M^*}{P^*}\right)^{-\rho}\left(\frac{w^*}{\theta}\right)]}{E[\left(\frac{M^*}{P^*}\right)^{-\rho}\left(\frac{1}{S}\right)]} = \frac{E[\left(\frac{M^*}{P^*}\right)^{-\rho}\left(\frac{w^*}{\theta}\right)p_F(i)^{-\mu}p_F^{\mu-1}M]+f^*}{E[\left(\frac{M^*}{P^*}\right)^{-\rho}\left(\frac{1}{S}\right)p_F(i)^{-\mu}p_F^{\mu-1}M]}$$

$$\frac{\alpha E[\left(\frac{M^*}{P^*}\right)^{-\rho}\left(\frac{\eta P^*}{\theta\left(\frac{M^*}{P^*}\right)^{-\rho}}\right)]}{E[\left(\frac{M^*}{P^*}\right)^{-\rho}\left(\frac{1}{S}\right)]} = \frac{E[\left(\frac{M^*}{P^*}\right)^{-\rho}\left(\frac{\eta P^*}{\theta\left(\frac{M^*}{P^*}\right)^{-\rho}}\right)p_F(i)^{-\mu}p_F^{\mu-1}M]+f^*}{E[\left(\frac{M^*}{P^*}\right)^{-\rho}\left(\frac{1}{S}\right)p_F(i)^{-\mu}p_F^{\mu-1}M]},$$

where $\alpha = \frac{\mu}{\mu-1}$, the markup. Since prices are set in advance, and treated by firms as given for goods other than their own, they can be extracted from the expectational operators:

$$\frac{\alpha\eta P^{*1-\rho}E[\frac{1}{\theta}]}{E[M^{*-\rho}\left(\frac{1}{S}\right)]} = \frac{\eta P^* p_F(i)^{-\mu}p_F^{\mu-1}E\left[\frac{M}{\theta}\right]+f^*}{P^{*\rho}p_F(i)^{-\mu}p_F^{\mu-1}E[M^{*-\rho}\left(\frac{1}{S}\right)M]}$$

$$\frac{\alpha E[\frac{1}{\theta}]}{E[M^{*-\rho}\left(\frac{1}{S}\right)]} = \left(\frac{f^*}{\eta P^* p_F(i)^{-\mu}p_F^{\mu-1}}+E\left[\frac{M}{\theta}\right]\right)\frac{1}{E[M^{*-\rho}\left(\frac{1}{S}\right)M]} \quad (27)$$

Assuming that all firms in each of the two industries have the same cost structure and face identical demand curves, $p_F(i)$ will be identical for all goods $i \in [0, n)$. By the same token, $p_H(i)$ will be identical for all goods $i \in [n, 1]$. It follows that

$$p_H = \left[\left(\frac{1}{n} \right) \int_0^n p_H(i)^{1-\mu} di \right]^{\frac{1}{1-\mu}} = \left[\left(\frac{1}{n} \right) (n)\, p_H(i)^{1-\mu} \right]^{\frac{1}{1-\mu}} = p_H(i) \quad (28)$$

and

$$p_F = \left[\left(\frac{1}{1-n} \right) \int_n^1 p_F(i)^{1-\mu} di \right]^{\frac{1}{1-\mu}} = \left[\left(\frac{1}{1-n} \right) (1-n)\, p_F(i)^{1-\mu} \right]^{\frac{1}{1-\mu}} = p_F(i). \quad (29)$$

Substituting the relation in (29) for p_F, (27) can be written

$$\begin{aligned}
\frac{\alpha E[\frac{1}{\theta}]}{E[M^{*-\rho}\left(\frac{1}{S}\right)]} &= \frac{f^* p_F(i)}{\eta P^* E[M^{*-\rho}\left(\frac{1}{S}\right) M]} + \frac{E\left[\frac{M}{\theta}\right]}{E[M^{*-\rho}\left(\frac{1}{S}\right) M]} \\
\frac{\alpha E[\frac{1}{\theta}] E[M^{*-\rho}\left(\frac{1}{S}\right) M]}{E[M^{*-\rho}\left(\frac{1}{S}\right)]} &= \frac{f^* p_F(i)}{\eta P^*} + E\left[\frac{M}{\theta}\right] \\
p_F(i) &= \left(\frac{\eta}{f^*}\right) \left(\frac{\alpha E[\frac{1}{\theta}] E[M^{*-\rho}\left(\frac{1}{S}\right) M]}{E[M^{*-\rho}\left(\frac{1}{S}\right)]} - E\left[\frac{M}{\theta}\right] \right) P^*.
\end{aligned}$$

Letting $q_1 = \frac{\alpha E[\frac{1}{\theta}] E[M^{*-\rho}\left(\frac{1}{S}\right) M]}{E[M^{*-\rho}\left(\frac{1}{S}\right)]} - E\left[\frac{M}{\theta}\right]$,

$$p_F(i) = \left(\frac{\eta}{f^*}\right) q_1 P^*. \quad (30)$$

Similarly, one can derive an equation relating the price of the home good in the foreign market, $p_H^*(i)$, to the home price level

$$p_H^*(i) = \left(\frac{\eta}{f}\right) q_2 P, \tag{31}$$

where $q_2 = \frac{\alpha E[\frac{1}{\theta^*}]E[M^{-\rho}(S)M^*]}{E[M^{-\rho}(S)]} - E\left[\frac{M^*}{\theta^*}\right]$.

The unrestriced pricing rule for $p_F^*(i)$ can be reduced to a function of $p_H^*(i)$, allowing the foreign price level, P^*, to be written as a function of $p_H^*(i)$. In the same way, the home price level, P, can be written as a function of $p_F(i)$. This is useful because expressing the foreign and home price levels as functions only of $p_H^*(i)$ and $p_F(i)$, respectively, and combining them with (30) and (31) yields two equations with two unknowns, allowing one to solve for both $p_F(i)$ and $p_H^*(i)$. The pricing rule for $p_F^*(i)$, which is not subject to entry conditions, is

$$p_F^*(i) = \left(\frac{\mu}{\mu - 1}\right) \frac{E[U_{c^*} \frac{w^*}{\theta^*}]}{E[U_{c^*}]},$$

which (substituting for wage and consumption variables) reduces to a function of $p_H^*(i)$,

$$p_F^*(i) = \left[\left(\frac{\alpha\eta E[\frac{1}{\theta^*}]}{E[M^{*-\rho}]}\right) p_H^*(i)^{n(1-\rho)}\right]^{\frac{1}{1-(1-n)(1-\rho)}}. \tag{32}$$

The foreign price level is therefore

$$\begin{aligned} P^* &= \left[\left(\frac{\alpha\eta E[\frac{1}{\theta^*}]}{E[M^{*-\rho}]}\right) p_H^*(i)^{n(1-\rho)}\right]^{\frac{1-n}{1-(1-n)(1-\rho)}} p_H^*(i)^n \\ &= \left(\frac{\alpha\eta E[\frac{1}{\theta^*}]}{E[M^{*-\rho}]}\right)^{\frac{1-n}{1-(1-n)(1-\rho)}} p_H^*(i)^{\frac{n}{1-(1-n)(1-\rho)}} \end{aligned} \tag{33}$$

Repeating the process, the unrestricted pricing rule for $p_H(i)$ is

$$p_H(i) = \left[\left(\frac{\alpha\eta E[\frac{1}{\theta}]}{E[M^{-\rho}]}\right) p_F(i)^{(1-n)(1-\rho)}\right]^{\frac{1}{1-n(1-\rho)}} \tag{34}$$

and P can be expressed

$$P = \left(\frac{\alpha\eta E[\frac{1}{\theta}]}{E[M^{-\rho}]}\right)^{\frac{n}{1-n(1-\rho)}} p_F(i)^{\frac{1-n}{1-n(1-\rho)}} \tag{35}$$

Using (30) and (31) in combination with these expressions for the home and domestic price levels, one can produce two equations which can be solved for $p_F(i)$ and $p_H^*(i)$:

$$p_F(i) = \left(\frac{\eta}{f^*}\right) q_1 \left(\frac{\alpha\eta E[\frac{1}{\theta^*}]}{E[M^{*-\rho}]}\right)^{\frac{1-n}{1-(1-n)(1-\rho)}} p_H^*(i)^{\frac{n}{1-(1-n)(1-\rho)}} \tag{36}$$

$$p_H^*(i) = \left(\frac{\eta}{f}\right) q_2 \left(\frac{\alpha\eta E[\frac{1}{\theta}]}{E[M^{-\rho}]}\right)^{\frac{n}{1-n(1-\rho)}} p_F(i)^{\frac{1-n}{1-n(1-\rho)}}. \tag{37}$$

Substituting (36) into (35) yields a closed, pseudo-reduced form for $p_F(i)$,

$$p_F(i) = \left[\frac{\eta k_1 q_1}{f^*}\left(\frac{\eta k_2 q_2}{f}\right)^{\beta}\right]^{\frac{1}{1-\beta\gamma}},$$

for $k_1 = \left(\frac{\alpha\eta E[\frac{1}{\theta}]}{E[M^{*-\rho}]}\right)^{\frac{1-n}{1-(1-n)(1-\rho)}}$, $k_2 = \left(\frac{\alpha\eta E[\frac{1}{\theta^*}]}{E[M^{-\rho}]}\right)^{\frac{n}{1-n(1-\rho)}}$, $\beta = \frac{n}{1-(1-n)(1-\rho)}$, and $\gamma = \frac{1-n}{1-n(1-\rho)}$.

Substituting $S = \frac{M}{M^*}$, the pseudo-reduced form under floating exchange rates becomes

$$p_F(i) = \left\{\frac{\eta k_1}{f^*}\left[\frac{\alpha E[\frac{1}{\theta}]E[M^{*1-\rho}]}{E[M^{*1-\rho}\left(\frac{1}{M}\right)]} - E[\frac{M}{\theta}]\right]\left[\frac{\eta k_2}{f}\left(\frac{\alpha E[\frac{1}{\theta^*}]E[M^{1-\rho}]}{E[M^{1-\rho}\left(\frac{1}{M^*}\right)]} - E[\frac{M^*}{\theta^*}]\right)\right]^{\beta}\right\}^{\frac{1}{1-\beta\gamma}}.$$

If the home and foreign money supplies are independently distributed, the expression further reduces to

$$p_F(i) = \left\{\frac{\eta k_1}{f^*}\left[\frac{\alpha E[\frac{1}{\theta}]}{E[\left(\frac{1}{M}\right)]} - E[\frac{M}{\theta}]\right]\left[\frac{\eta k_2}{f}\left(\frac{\alpha E[\frac{1}{\theta^*}]}{E\left(\frac{1}{M^*}\right)]} - E[\frac{M^*}{\theta^*}]\right)\right]^{\beta}\right\}^{\frac{1}{1-\beta\gamma}}.$$

At this point, a reduced-form solution can be found by assuming that θ, θ^*, M and M^* are independently and lognormally distributed as specified in the main text. Then we have

$$p_F(i) \quad = \quad \left(\frac{\eta}{f^*}\right)^{\frac{1}{1-\beta\gamma}}\left(\alpha\eta e^{\rho m^* - \bar{\theta}^* - \frac{1}{2}\rho(\rho+1)\sigma^2_{m^*} + \sigma^2_{\theta^*}}\right)^{\delta}\left(\frac{\eta}{f}\right)^{\beta\epsilon}\left(\alpha\eta e^{\rho m - \bar{\theta} - \frac{1}{2}\rho(\rho+1)\sigma^2_m + \sigma^2_{\theta}}\right)^{\frac{\beta\epsilon}{1-\beta\gamma}}$$

$$\times \left(\alpha e^{m^*-\bar{\theta}^*-\sigma_{m^*}^2+\sigma_{\theta^*}^2} - e^{m^*-\bar{\theta}^*+\sigma_{\theta^*}^2}\right)^{\frac{\beta}{1-\beta\gamma}} \left(\alpha e^{m-\bar{\theta}-\sigma_m^2+\sigma_{\theta^*}^2} - e^{m-\bar{\theta}+\sigma_\theta^2}\right)^{\frac{1}{1-\beta\gamma}},$$

where $\delta = \frac{1-n}{1-(1-n)(1-\rho)}$ and $\epsilon = \frac{n}{1-n(1-\rho)}$.

To focus on the influence of the foreign money supply on the home-country price level, productivity parameters are extracted from the following analysis. Factoring and taking logs, an expression for the log price of the foreign good in the home country emerges:

$$\begin{aligned}\log(p_F(i)) &= \frac{1}{1-\beta\gamma}[\log z + (\delta\rho+\beta)m^* - \delta\frac{1}{2}\rho(\rho+1)\sigma_{m^*}^2 + (\beta\epsilon\rho+1)m \\ &\quad -\beta\epsilon\frac{1}{2}\rho(\rho+1)\sigma_m^2 + \log(\alpha e^{-\sigma_{m^*}^2} - 1) + \log(\alpha e^{-\sigma_m^2} - 1)].\end{aligned}$$